EE-557
Semiconductor devices I

Continuity and Shockley equations

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Master Program
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Outline of the lecture

- Continuity equations
- Shockley equations
- Examples and exercises

Read Chapter 5 (from Shockley equations) of the reference book (on moodle)

References:

- J. A. del Alamo, course materials for 6.720J Integrated Microelectronic Devices, Spring 2007. MIT OpenCourseWare (http://ocw.mit.edu/)

So far we have learned:

- Recombination
- Generation
- Drift
- Diffusion

How to describe carrier flow in semiconductors taking into account all these mechanisms?

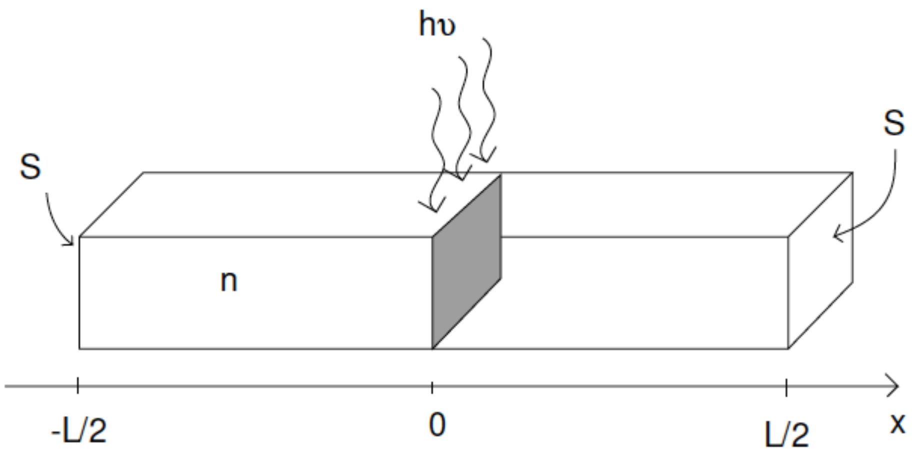
Carrier flow

Carrier dynamics:

$$\frac{dn}{dt} = \frac{dp}{dt} = G - R$$

Applies only to uniform problems in space!

This doesn't work in problems like this: Generation of carriers in a given position of space



Equation system does not capture:

- **Impact of carrier movement** on carrier concentration
  (i.e. when carriers move away from a point, their concentration drops!)

- **Boundary conditions** (surfaces are not infinitely far away – covered in Del Alamo-Chapter 5)

We need to include the flow of carriers

Continuity equations:

For electrons: $$\frac{\partial n}{\partial t} = G - R + \frac{1}{q}\vec{\nabla}.\vec{J}_e$$

For holes: $$\frac{\partial p}{\partial t} = G - R - \frac{1}{q}\vec{\nabla}.\vec{J}_h$$

Semiconductor physics so far: Shockley's equations

Gauss' law: $\vec{\nabla} \cdot \vec{\mathcal{E}} = \frac{q}{\epsilon}(p - n + N_D^+ - N_A^-)$

Electron current equation: $\vec{J}_e = -qn\vec{v}_e^{drift} + qD_e\vec{\nabla}n$

Hole current equation: $\vec{J}_h = qp\vec{v}_h^{drift} - qD_h\vec{\nabla}p$

Electron continuity equation: $\frac{\partial n}{\partial t} = G_{ext} - U(n,p) + \frac{1}{q}\vec{\nabla} \cdot \vec{J}_e$

Hole continuity equation: $\frac{\partial p}{\partial t} = G_{ext} - U(n,p) - \frac{1}{q}\vec{\nabla} \cdot \vec{J}_h$

Total current equation: $\vec{J}_t = \vec{J}_e + \vec{J}_h$

System of non-linear, coupled partial differential equations: generally not solvable in a closed form.

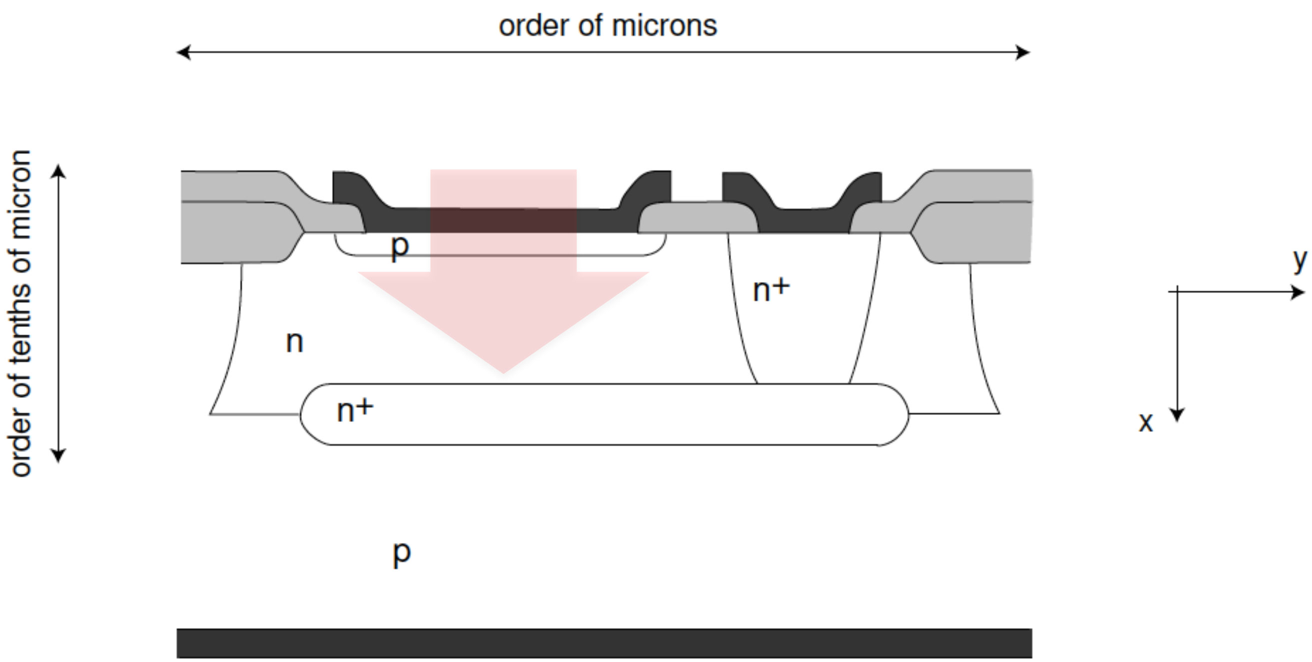
*** Attention: Here the net recombination rate *U(n,p)* includes all the absolute net recombination rates, including surface recombination, which were not discussed in this course.

One-dimensional approximation

1D approximation: $\vec{\nabla} \Rightarrow \frac{\partial}{\partial x}$

In many cases, complex problems can be broken into several 1D sub-problems.

Example: integrated p-n diode



Shockley equations in 1D quasi-neutral situations

in 1D:

Gauss' law: $\frac{\partial \mathcal{E}}{\partial x} = \frac{q}{\epsilon}(p - n + N_D - N_A)$

Electron current equation: $J_e = -qnv_e^{drift}(\mathcal{E}) + qD_e\frac{\partial n}{\partial x}$

Hole current equation: $J_h = qpv_h^{drift}(\mathcal{E}) - qD_h\frac{\partial p}{\partial x}$

Electron continuity equation: $\frac{\partial n}{\partial t} = G_{ext} - U(n,p) + \frac{1}{q}\frac{\partial J_e}{\partial x}$

Hole continuity equation: $\frac{\partial p}{\partial t} = G_{ext} - U(n,p) - \frac{1}{q}\frac{\partial J_h}{\partial x}$

Total current equation: $J_t = J_e + J_h$

Equation set difficult because of coupling through Gauss' law

We can identify two contributions:

$$\rho = q(p - n + N_D^+ - N_A^-) = q(p_o - n_o + N_D^+ - N_A^-) + q(p' - n')$$

$$\mathcal{E} = \mathcal{E}_o + \mathcal{E}'$$

$$\frac{\partial \mathcal{E}_o}{\partial x} = \frac{q}{\epsilon}(p_o - n_o + N_D^+ - N_A^-)$$ electric field in equilibrium

$$\frac{\partial \mathcal{E}'}{\partial x} = \frac{q}{\epsilon}(p' - n')$$ Excess electric field (out of equilibrium) e.g. externally applied voltage

Shockley equations in 1D quasi-neutral situations

Two broad classes of important situations to break Gauss' law coupling:

1. Carrier concentrations are high: **quasi-neutral situation**
   - majority carrier concentration nearly tracks the doping concentration
   - Any introduction of extra carriers in a volume is negligible compared to the charge density already present

$$\rho \simeq 0 \;\Rightarrow\; \frac{\partial \mathcal{E}}{\partial x} \simeq 0$$

2. Carrier concentrations are very low: **space-charge** and **high-resistivity situations**: **E independent of n, p**

Simplifications of Shockley equations to 1D quasi-neutral situations

Quasi-neutral (QN) approximation
At every location, the net volume charge that arises from a discrepancy of the concentration of positive and negative species is **negligible** in the scale of the charge density that is present.

$$\rho \simeq 0 \;\Rightarrow\; \frac{\partial \mathcal{E}}{\partial x} \simeq 0$$

QN approximation eliminates Gauss' law from the set:

$$\rho = q(p - n + N_D^+ - N_A^-) = q(p_o - n_o + N_D^+ - N_A^-) + q(p' - n')$$

Quasi-neutrality out of equilibrium:

$$|\frac{p' - n'}{n'}| \simeq |\frac{p' - n'}{p'}| \ll 1$$

$$p' \simeq n'$$

QN approximation is good if **n, p high** ⇒carriers move to erase ρ.

Shockley equations in quasi-neutrality

$$p - n + N_{\mathrm{D}} - N_{\mathrm{A}} \simeq 0$$

$$J_{\mathrm{e}} = -qnv_{\mathrm{e}}^{\mathrm{drift}} + qD_{\mathrm{e}}\frac{\partial n}{\partial x}$$

$$J_{\mathrm{h}} = qpv_{\mathrm{h}}^{\mathrm{drift}} - qD_{\mathrm{h}}\frac{\partial p}{\partial x}$$

$$\frac{\partial n}{\partial t} = G_{\mathrm{ext}} - U + \frac{1}{q}\frac{\partial J_{\mathrm{e}}}{\partial x} \quad \text{or} \quad \frac{\partial p}{\partial t} = G_{\mathrm{ext}} - U - \frac{1}{q}\frac{\partial J_{\mathrm{h}}}{\partial x}$$

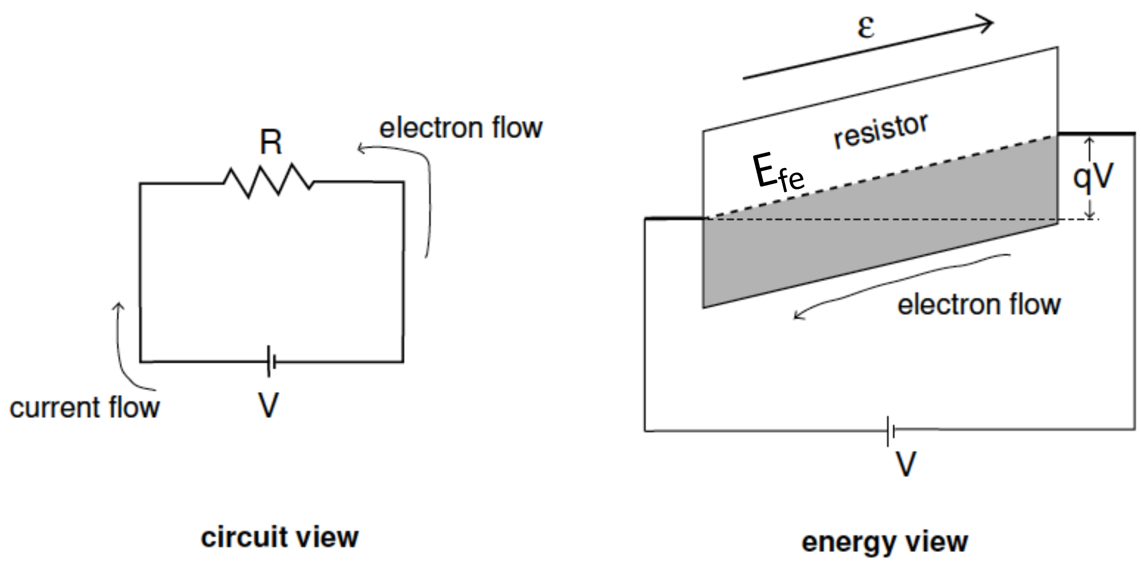
$$\frac{\partial J_{\mathrm{t}}}{\partial x} \simeq 0$$

$$J_{\mathrm{t}} = J_{\mathrm{e}} + J_{\mathrm{h}}$$

Case 1: Majority-carrier type situations

Semiconductor resistor

Voltage applied to extrinsic quasi-neutral semiconductor, without changing the equilibrium carrier concentrations:



The battery picks up electrons from positive terminal, increases their potential energy and puts them at the negative terminal.

Energy given to electrons: **E = qV**

If provided with a path (resistor), electrons flow.

Case 1: Majority-carrier type situations

Semiconductor resistor

Characteristics of majority carrier-type situations:

- Small electric field imposed from outside: does not disturb the dynamic balance existing in equilibrium
- Carriers are not disturbed from equilibrium: time derivatives are zero!
- electrons and holes drift $\frac{dJ_e}{dx} \simeq 0, \frac{dJ_h}{dx} \simeq 0, \frac{dJ_t}{dx} \simeq 0$

Electron and hole concentrations unperturbed from TE Simplifications:

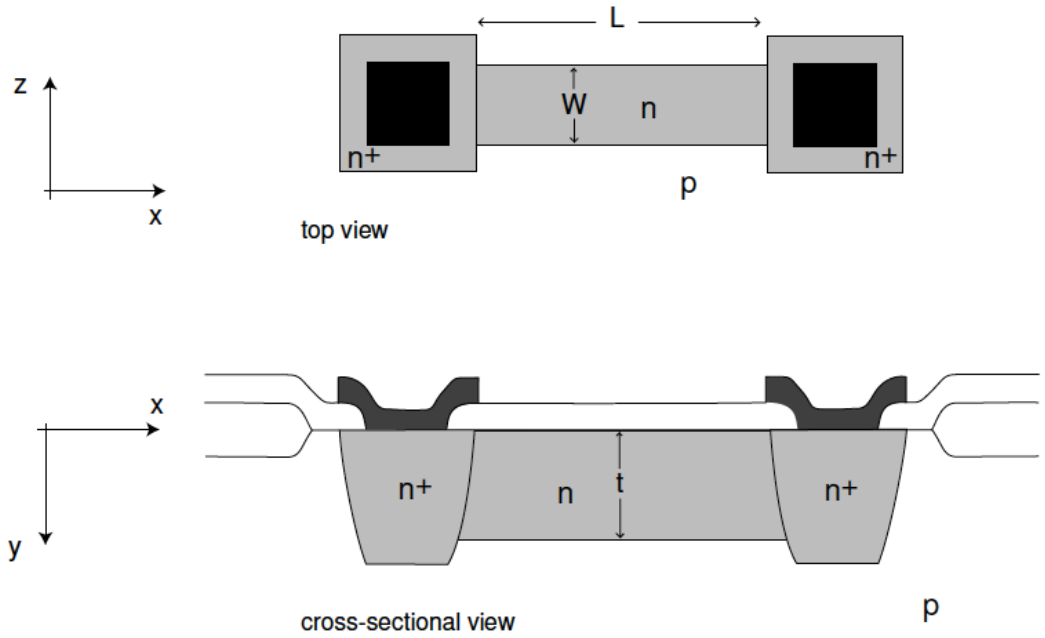
- neglect contribution of minority carriers
- neglect time derivatives of carrier concentrations

⇒ problem becomes completely quasi-static
Equation set for 1D majority-carrier type situations:

n-type	p-type
$n \simeq n_o \simeq N_D$	$p \simeq p_o \simeq N_A$
$J_e = -qn_o[v_{de}(\mathcal{E}) - v_{de}(\mathcal{E}_o)]$	$J_h = qp_o[v_{dh}(\mathcal{E}) - v_{dh}(\mathcal{E}_o)]$
$\frac{dJ_e}{dx} \simeq 0, \frac{dJ_h}{dx} \simeq 0, \frac{dJ_t}{dx} \simeq 0$	
$J_t \simeq J_e$	$J_t \simeq J_h$

Simplifications of Shockley equations to 1D quasi-neutral situations

Integrated Resistor with uniform doping (n-type)



Equations:

$$J_t = -qN_D v_e^{drift}(\mathcal{E})$$

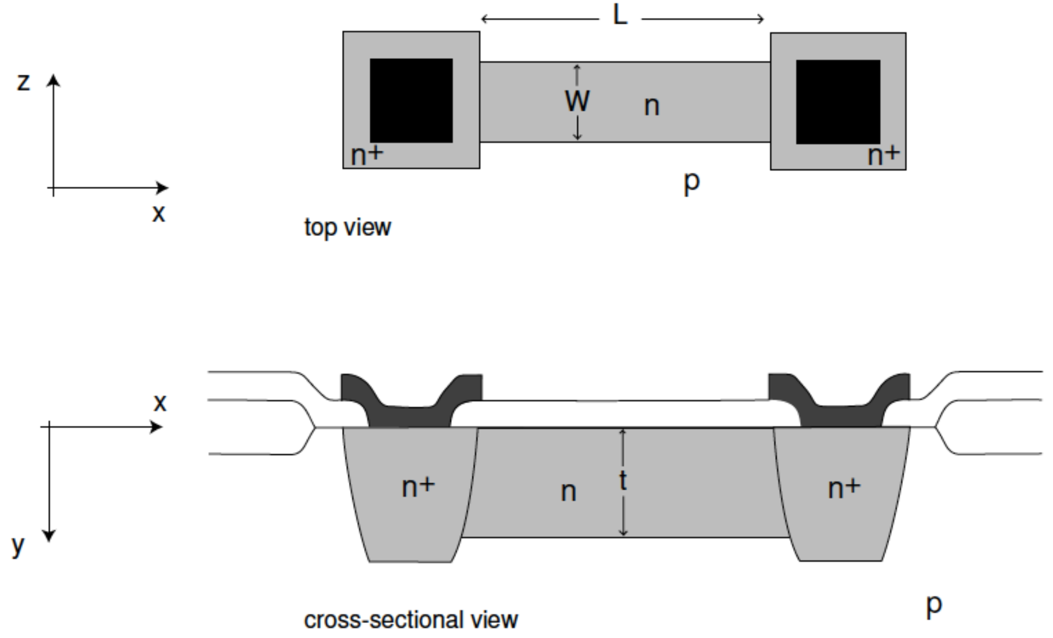
$$J_t \simeq qN_D \mu_e \mathcal{E}$$

$$I = WtqN_D \mu_e \frac{V}{L}$$

In general (low and high fields):

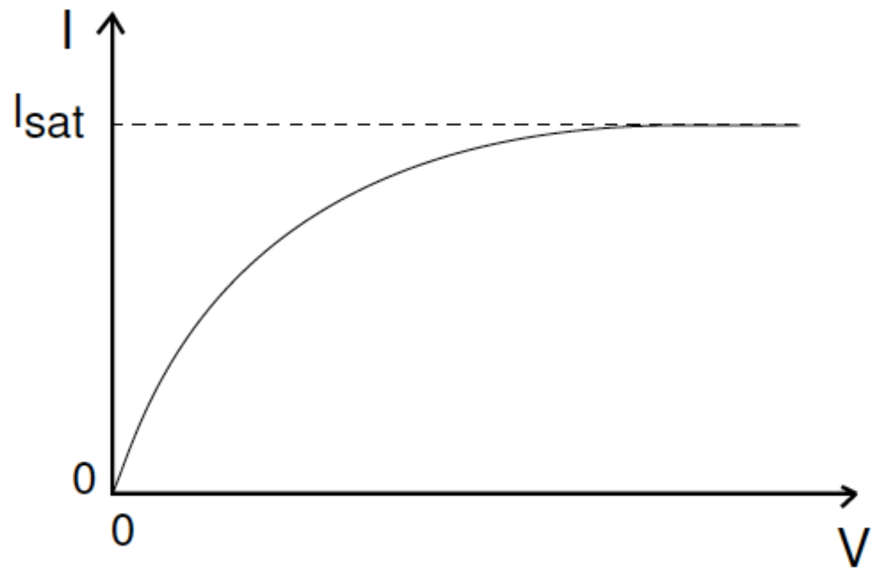
$$I = WtqN_D \frac{v_{sat}}{1 + \frac{v_{sat}}{\mu_e}\frac{L}{V}}$$

Integrated Resistor with uniform doping (n-type)



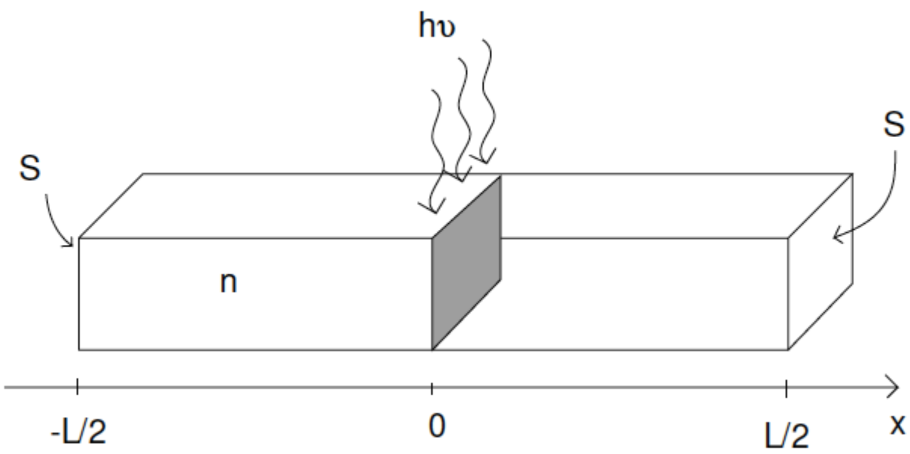
which for high fields saturates to:

$$I_{sat} = WtqN_D v_{sat}$$



Case 2: Minority-carrier type situations

Characterized by (i) quasi-neutrality, (ii) presence of excess carriers as a result of external generation or injection from an adjacent region, and (iii) absence of a significant external electric field.



Considering low level injection:

• majority carrier concentrations out of equilibrium are basically unchanged: in n-type material n~no.

• minority carrier concentrations are overwhelmed. In an n-type semiconductor, p~po

• recombination rate is proportional to excess carriers over carrier lifetime. For n-type, U = p'/τ

• Even though no external fields are applied, internal fields can be generated as a result of carrier injection. However, this is our fourth simplification, and internally generated electric fields are small enough so that minority carrier drift currents produced by them are insignificant

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<table>
<tr><th>n-type</th><th>p-type</th></tr>
<tr><td colspan="2">$p_o - n_o + N_D - N_A \simeq 0$</td></tr>
<tr><td colspan="2">$p' \simeq n'$</td></tr>
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<tr><td>$D_h\frac{\partial^2 p'}{\partial x^2} - \mu_h\mathcal{E}_o\frac{\partial p'}{\partial x} - \frac{p'}{\tau} + G_{ext} = \frac{\partial p'}{\partial t}$</td><td>$D_e\frac{\partial^2 n'}{\partial x^2} + \mu_e\mathcal{E}_o\frac{\partial n'}{\partial x} - \frac{n'}{\tau} + G_{ext} = \frac{\partial n'}{\partial t}$</td></tr>
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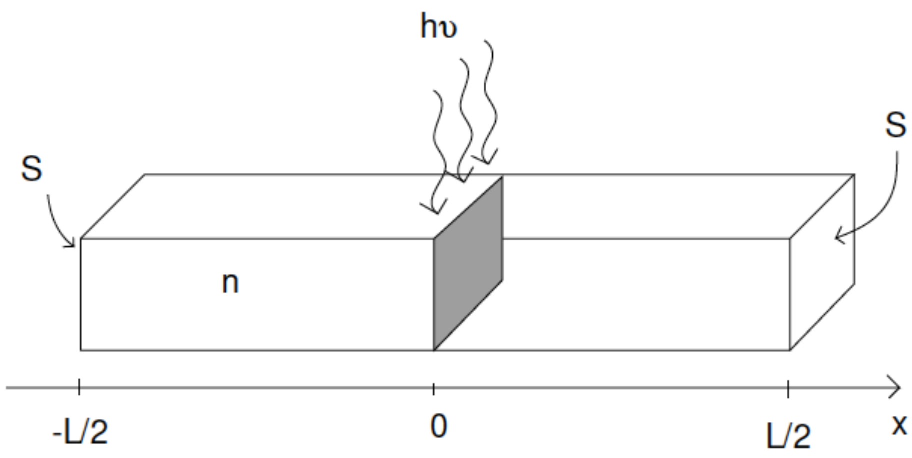
Case 2: Minority-carrier type situations

- Let's look at a n-type bar



Quiz

1. In the context of carrier transport, the continuity equation primarily expresses:
A. The relationship between drift and diffusion currents under steady-state conditions.
B. Conservation of total charge within a differential semiconductor volume.
C. The local electric field due to space charge in quasi-neutral regions.
D. The recombination-generation balance in equilibrium.

2. The **Shockley equations** combine which fundamental physical laws?
A. Poisson's equation, Ohm's law, and Einstein's relation.
B. Gauss's law, current continuity, and drift–diffusion relations.
C. Fick's law, charge neutrality, and carrier lifetime relations.
D. Boltzmann statistics, recombination rate, and the diffusion equation.

3. In a **majority carrier situation**, such as current flow in an n-type resistor:
A. Carrier drift dominates, and quasi-neutrality ensures $\nabla \cdot J = 0$.
B. Space charge dominates, and the electric field is nonuniform.
C. Diffusion dominates, and the total current density is zero.
D. Both drift and diffusion vanish in steady state.

4. In a "long" bar under localized illumination, the excess minority carrier profile decays exponentially with distance because:
A. The diffusion coefficient decreases with position.
B. The carrier lifetime varies with the doping level.
C. The balance between diffusion and recombination leads to an exponential steady-state solution.
D. The internal electric field pulls carriers uniformly across the sample.

Shockley equations:
system of equations that describes carrier phenomena in semiconductors in the drift-diffusion regime.

Quasi-neutral approximation:
appropriate if semiconductor is sufficiently extrinsic: ρ ~ 0 ⇒

$$n_o - p_o \simeq N_D - N_A \qquad\qquad n' \simeq p'$$

Majority carrier-type situations characterized by application of **external voltage without perturbing carrier concentrations**.

Majority-carrier type situations dominated by drift of majority carriers.
Integrated resistor:

- for low voltages, current proportional to voltage across
- for high voltages, current saturates due to v^{sat}